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LETTER TO THE EDITOR

Curvature, zero modes and quantum statistics

M Calixto[1,2] **and V Aldaya**[2]

[1] Departamento de Matemática Aplicada y Estadística, Universidad Politécnica de Cartagena, Paseo Alfonso XIII 56, 30203 Cartagena, Spain
[2] Instituto de Astrofísica de Andalucía, Apartado Postal 3004, 18080 Granada, Spain

E-mail: Manuel.Calixto@upct.es and valdaya@iaa.es

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Abstract

We explore an intriguing connection between the Fermi–Dirac and Bose–Einstein statistics and the thermal baths obtained from a vacuum radiation of coherent states of zero modes in a second quantized (many-particle) theory on the compact $O(3)$ and noncompact $O(2, 1)$ isometry subgroups of the de Sitter and anti-de Sitter spaces, respectively. The high frequency limit is retrieved as a (zero-curvature) group contraction to the Newton–Hooke (harmonic oscillator) group. We also make some comments on the vacuum energy density and the cosmological constant problem.

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1. Introduction

The spin-statistics theorem in quantum field theory relates the spin of a particle to the statistics obeyed by that particle. Here we investigate an interesting correspondence between curvature ('boundness' and compactness) and quantum statistics. The interrelation between both concepts is established through vacuum coherent configurations of zero modes in quantum field theory.

Quantum vacua are not really empty. We know that zero-point energy, like other non-zero vacuum expectation values, leads to observable consequences such as, for instance, the Casimir effect [1], and influences the behaviour of the universe at cosmological scales, where the vacuum (dark) energy is expected to contribute to the cosmological constant, which affects the expansion of the universe. In quantum field theory, one expects the vacuum state to be stable under the basic symmetry transformations G (namely, the Poincaré group). Then the action of some spontaneously broken symmetry transformations can destabilize the vacuum and make it to *radiate*. Such is the case of the Planckian radiation of the Poincaré-invariant vacuum under uniform accelerations, that is, the Unruh effect [2]. Here, the Poincaré-invariant vacuum looks the same to any inertial observer but converts into a thermal bath of radiation with temperature

$T = \hbar a/2\pi c k_B$ in passing to a uniformly accelerated frame (a denotes the acceleration, c the speed of light and k_B the Boltzmann constant). These radiation phenomena are usually linked to some kind of global mutilation of the spacetime (namely, existence of horizons). In [3], it was shown that the reason for this radiation is more profound and related to the spontaneous breakdown of the conformal symmetry in quantum field theory. From this point of view, a Poincaré-invariant vacuum can be regarded as a coherent state of conformal zero modes, which are undetectable ('dark') by inertial observers but unstable under relativistic uniform accelerations (special conformal transformations). There we used the conformal group in (1+1) dimensions, $SO(2,2) \simeq SO(2,1) \times SO(2,1)$, which consists of two copies of the pseudo-orthogonal group $SO(2,1)$ (left- and right-moving modes, respectively).

In this letter, we construct $O(3)$, $O(2,1)$ and Newton–Hooke-invariant quantum field theories in a unified manner. We could think of $O(3)$ and $O(2,1)$ as isometry subgroups of the spatial part of the de Sitter and anti-de Sitter spaces, with positive and negative curvature κ, respectively. We shall work with their double covers, $U(2)$ and $U(1,1)$ instead, for convenience. The Lie algebra commutators of our basic symmetry group G_κ will be

$$[A_+, A_-] = 2\kappa H - \Xi, \qquad [H, A_\pm] = \pm A_\pm, \qquad [\Xi, \text{all}] = 0, \tag{1}$$

where H will represent the Hamiltonian, Ξ will play the role of the zero-point energy (or the total number of particles operator in second quantization), $A_\pm$ will be *ladder* creation and annihilation operators and $\kappa = \pm 1, 0$ is the *curvature* parameter for $U(2)$, $U(1,1)$ and the Newton–Hooke (harmonic oscillator) groups, respectively.

The group $U(1,1)$ is noncompact, so, unlike the case of $U(2)$, all its unitary irreducible representations (unirreps) are infinite dimensional. This group has a number of series of unirreps: principal, discrete and supplementary. We shall consider here only representations of the discrete series where each carrier space $\mathcal{H}_s$ is labelled by the (*conformal*) *spin* $s = 1/2, 1, 3/2, 2, \ldots$ and is spanned by the orthonormal basis $B(\mathcal{H}_s) = \{|s,n\rangle, n = 0, 1, 2, \ldots\}$. The action of the operators (1) on these basis vectors is

$$\begin{aligned} H|s,n\rangle &= n|s,n\rangle, & A_+|s,n\rangle &= \sqrt{(n+1)(2s-\kappa n)}|s,n+1\rangle, \\ \Xi|s,n\rangle &= 2s|s,n\rangle, & A_-|s,n\rangle &= \sqrt{n(2s-\kappa(n-1))}|s,n-1\rangle. \end{aligned} \tag{2}$$

Looking at this representation, we can think of an 'exotic' harmonic oscillator with an equispaced energy spectrum $E_n = \epsilon n$, where we have introduced the interlevel energy spacing $\epsilon = \hbar\omega$ to give dimensions. For negative, $\kappa = -1$, and zero, $\kappa = 0$, curvature (i.e., for $U(1,1)$ and the Newton–Hooke groups, respectively), this energy spectrum is unbounded from above, whereas for positive curvature, $\kappa = 1$ (i.e., for $U(2)$), we have a bounded spectrum with $2s+1$ states.

Using the standard Iwasawa decomposition (see e.g. [4]), any group element $U \in G_\kappa$ can be represented as

$$U(\zeta, \bar\zeta, \tau, \varphi) = \mathrm{e}^{\zeta A_+ - \bar\zeta A_-}\, \mathrm{e}^{\mathrm{i}\tau H}\, \mathrm{e}^{\mathrm{i}\varphi\Xi}, \tag{3}$$

where $\varphi, \tau \in [0, 2\pi]$ and $\zeta \in \mathbb{C}$. An important ingredient to construct a G_κ-invariant quantum field theory, as a second quantization on G_κ, will be the irreducible matrix coefficients of representation (2) in the orthonormal basis $B(\mathcal{H}_s)$:

$$U^{(s)}_{mn}(\zeta, \bar\zeta, \tau, \varphi) \equiv \langle s, m|U(\zeta, \bar\zeta, \tau, \varphi)|s, n\rangle. \tag{4}$$

Given the Fourier expansion in modes of a field with (conformal) spin s,

$$|\psi\rangle = \sum_{n=0} a_n |s, n\rangle, \tag{5}$$

the Fourier coefficients a_n (resp. $a_n^\dagger$) are promoted to annihilation (resp. creation) operators of energy modes $E_n = \hbar\omega n$ in the second quantized theory, with commutation relations $\left[a_n, a_m^\dagger\right] = \delta_{n,m}$. The (finite) action of G_κ on annihilation operators is

$$a_m \to a'_m = \sum_{n=0}^{\infty} U_{mn}^{(s)}(\zeta, \bar\zeta, \tau, \varphi) a_n, \tag{6}$$

together with the conjugated expression for creation operators. The infinitesimal generators of this finite action are the second quantized version, $\hat H$, $\hat\Xi$, $\hat A_\pm$, of the basic operators (1). They have the following expression:

$$\begin{aligned} \hat H &= \sum_n n a_n^\dagger a_n, & \hat A_+ &= \sum_n \sqrt{(n+1)(2s-\kappa n)}\, a_n^\dagger a_{n+1}, \\ \hat\Xi &= 2s \sum_n a_n^\dagger a_n, & \hat A_- &= \sum_n \sqrt{n(2s-\kappa(n-1))}\, a_n^\dagger a_{n-1}, \end{aligned} \tag{7}$$

where summations start at $n = 0$ and are finite or infinite, depending on the curvature κ. Here we highlight the *total energy* operator, $\hat E \equiv \hbar\omega \hat H$, and the *total number of particles* operator, $\hat N \equiv \frac{1}{2s}\hat\Xi$.

The vacuum $|0\rangle$ of the second quantized theory is characterized by being stable under the basic isometry group G_κ, i.e.

$$U|0\rangle = |0\rangle, \qquad \forall U \in G_\kappa \quad \Rightarrow \quad \{\hat H, \hat\Xi, \hat A_+, \hat A_-\}|0\rangle = 0, \tag{8}$$

and annihilated by $a_n|0\rangle = 0, \forall n \geqslant 0$. Then an orthonormal basis for the Hilbert space of the second quantized theory is constructed by taking the orbit through the vacuum $|0\rangle$ of the creation operators $a_n^\dagger$:

$$|q(n_1), \ldots, q(n_p)\rangle \equiv \frac{\left(a_{n_1}^\dagger\right)^{q(n_1)} \cdots \left(a_{n_p}^\dagger\right)^{q(n_p)}}{(q(n_1)! \cdots q(n_p)!)^{1/2}} |0\rangle, \tag{9}$$

where $q(n) \in \mathbb{N}$ denotes the occupation number of the energy level n. Note that any second quantized state (9) made up of an arbitrary content of *zero modes*, like

$$|\theta\rangle \equiv \sum_{q=0}^{\infty} \theta_q \left(a_0^\dagger\right)^q |0\rangle, \tag{10}$$

has zero total energy, i.e. $\hat H|\theta\rangle = 0$. It also verifies $\hat A_-|\theta\rangle = 0$ and $a_n|\theta\rangle = 0, \forall n > 0$, so that the state (10) behaves as a (degenerated) vacuum under the (unbroken) subgroup $B \subset G_\kappa$ of affine or similitude transformations, generated by $\mathcal{B} = \{\hat H, \hat A_-\}$. Moreover, given that a_0 acts as (a multiple of) the identity operator in the broken theory, that is, it commutes with

$$[a_0, \hat H] = [a_0, \hat A_-] = \left[a_0, a_n^\dagger\right] = 0, \qquad \forall n > 0, \tag{11}$$

it is natural to demand a_0 to leave the affine vacuum (10) stable, which implies that

$$a_0|\theta\rangle = \theta|\theta\rangle \quad \Rightarrow \quad |\theta\rangle = \mathrm{e}^{\theta a_0^\dagger - \bar\theta a_0}|0\rangle. \tag{12}$$

Thus, the vacuum of our (spontaneously) broken theory will be a *coherent state of zero modes* (see [4] and [5] for a thorough exposition on coherent states). The squared modulus of the complex parameter θ has the physical significance of the vacuum expectation value of the number operator in the new vacuum; indeed, one can verify that

$$\langle\theta|\hat N|\theta\rangle = |\theta|^2. \tag{13}$$

We can think of zero modes as *virtual* particles without ('bright') energy, undetectable ('dark') by affine observers. However, we shall show that a general unitary symmetry transformation

(6), which incorporates the broken symmetry generator $\hat{A}_+$, produces a 'rearrangement' of the the affine vacuum $|\theta\rangle$ and makes it to radiate. In other words, we shall associate a *thermal bath* with the excited (or, let us say, 'polarized') vacuum,

$$|\theta'\rangle \equiv \mathrm{e}^{\theta a_0'^\dagger - \bar{\theta} a_0'}|0\rangle, \tag{14}$$

and we shall show in sections 2 and 3 that the *mean energy per mode*, $\langle\theta'|\hat{E}|\theta'\rangle$, matches the Bose–Einstein (BE) and Fermi–Dirac (FD) expressions for the non-compact $U(1,1)$ and compact $U(2)$ isometry groups ($\kappa = \mp 1$), respectively. The limit of high frequencies (or large interlevel energy spacing $\hbar\omega \gg k_B T$) is regained from a contraction to the Newton–Hooke group (zero curvature, $\kappa = 0$), as explained in section 4. Note however that the average number of particles is conserved under this (unitary) transformation, i.e. $\langle\theta'|\hat{N}|\theta'\rangle = |\theta|^2$, because $U^\dagger U = 1$. We shall relate this quantity to a vacuum energy density, the sign of which depends on the curvature κ, and we shall make some comments on the cosmological constant problem in section 5.

2. Hyperbolic geometry and BE statistics

The irreducible matrix coefficients of representation (2) in the orthonormal basis $B(\mathcal{H}_s)$ are given in this case ($\kappa = -1$) by

$$U^{(s)}_{mn}(\zeta, \bar{\zeta}, \tau, \varphi) = \mathrm{e}^{2s\mathrm{i}\varphi}(1 - z\bar{z})^s \sqrt{\frac{C^{(s)}_m}{C^{(s)}_n}} \sum_{q=\max(0,m-n)}^{m} \binom{n}{m-q}\binom{2s+n+q-1}{q} \times (-1)^{n-m+q}\,\mathrm{e}^{\mathrm{i}n\tau} z^q \bar{z}^{n-m+q}, \tag{15}$$

where $C^{(s)}_n = n!/(2s+n-1)!$, $\zeta = |\zeta|\,\mathrm{e}^{\mathrm{i}\phi}$ and $z \equiv \mathrm{e}^{\mathrm{i}\phi}\tanh|\zeta|$ is restricted to the unit disc $D = \{z \in \mathbb{C} : |z| < 1\}$, which is the stereographic projection of the upper sheet of the hyperboloid $\mathbb{H}^2 = \{\vec{v} \in \mathbb{R}^3 : \vec{v}^2 = v_0^2 - v_1^2 - v_2^2 = 1, v_0 > 0\}$ onto the complex plane. The correspondence is established through

$$\vec{v} = (\cosh(2|\zeta|), \sinh(2|\zeta|)\cos\phi, \sinh(2|\zeta|)\sin\phi).$$

The finite action (6) of $U(1,1)$ on annihilation (resp. creation) operators of zero modes is

$$a_0 \to a_0' = \mathrm{e}^{2s\mathrm{i}\varphi}(1 + z\bar{z})^s \sum_{n=0}^{\infty}(-1)^n \binom{2s+n-1}{n}^{1/2} \mathrm{e}^{\mathrm{i}n\tau}\bar{z}^n a_n, \tag{16}$$

which leads to the excited vacuum:

$$|\theta'\rangle \equiv \mathrm{e}^{\theta a_0'^\dagger - \bar{\theta} a_0'}|0\rangle = \mathrm{e}^{-|\theta|^2/2} \sum_{q=0}^{\infty} z^q \sum_{\substack{m_1,\ldots,m_q:\\ \sum_{n=1}^q n m_n = q}} \prod_{n=0}^{q} \frac{R_n^{m_n}}{m_n!} \prod_{n=0}^{q} (a_n^\dagger)^{m_n}|0\rangle, \tag{17}$$

where $R_n \equiv \theta(-1)^n\,\mathrm{e}^{-2s\mathrm{i}\varphi - \mathrm{i}n\tau}(1 - z\bar{z})^s \binom{2s+n-1}{n}^{1/2}$ and we set $m_0 \equiv 0$. We have used the general identity

$$\left(\sum_{n=0}^{\infty}\gamma_n z^n\right)^l = \sum_{q=0}^{\infty}\delta_q z^q, \qquad \delta_0 = \gamma_0^l, \qquad \delta_q = \frac{1}{q\gamma_0}\sum_{s=1}^{q}(sm - q + s)\gamma_s\delta_{q-s}. \tag{18}$$

From (17), we see that the relative probability of observing a state with total energy $E_q = \hbar\omega q$ in the excited vacuum $|\theta'\rangle$ is

$$P_q = \Omega(E_q)(|z|^2)^q, \qquad \Omega(E_q) \equiv \sum_{\substack{m_1,\ldots,m_q:\\ \sum_{n=1}^q n m_n = q}} \prod_{n=0}^{q} \frac{|R_n|^{2m_n}}{m_n!}. \tag{19}$$

We can associate a thermal bath with this distribution function by noting that $\Omega(E_q)$ behaves as a relative weight proportional to the number of states with energy $E_q = \hbar\omega q$; the factor $(|z|^2)^q$ fits this weight properly to a temperature T as

$$(|z|^2)^q = \mathrm{e}^{q\log|z|^2} = \mathrm{e}^{-\frac{E_q}{k_B T}}, \qquad T \equiv -\frac{\hbar\omega}{k_B \log|z|^2}. \tag{20}$$

We could think of (conformal) $U(1,1)$ transformations as transitions to a uniformly relativistic accelerated frame (see e.g. [3]), so that $T = \frac{\hbar a}{2\pi c k_B}$ is the temperature associated with a given acceleration $a \equiv -2\pi\omega c/\log|z|^2$.

After some intermediate calculations, the expected value of the total energy $\hat{E}$ in the accelerated vacuum $|\theta'\rangle$ proves to be

$$\begin{aligned}\langle\theta'|\hat{E}|\theta'\rangle &= |\theta|^2(1-|z|^2)^{2s}\sum_{n=0}^{\infty}\hbar\omega n\binom{2s+n-1}{n}|z|^{2n} \\ &= 2s|\theta|^2\hbar\omega\frac{|z|^2}{1-|z|^2} = 2s|\theta|^2\frac{\hbar\omega\,\mathrm{e}^{-\hbar\omega/k_B T}}{1-\mathrm{e}^{-\hbar\omega/k_B T}},\end{aligned} \tag{21}$$

which is proportional to the *mean energy per mode* of the BE statistics. In D spatial dimensions, the number of states with frequency between ω and $\omega + \mathrm{d}\omega$ is proportional to ω^{D-1}. Thus, for $D = 3$, the spectral distribution of the radiation of the accelerated vacuum $|\theta'\rangle$ is *Planckian*, i.e. $|\theta'\rangle$ radiates as a *black body*.

Still, we could have introduced a *chemical potential* μ by setting $|z|^2 = \mathrm{e}^{(\mu-\hbar\omega)/k_B T}$ with the restriction $\mu < \hbar\omega$, in order to preserve the condition $|z| < 1$.

3. Spherical geometry and FD statistics

The irreducible matrix coefficients of representation (2) in the orthonormal basis $B(\mathcal{H}_s)$ are given in this case ($\kappa = 1$) by

$$\begin{aligned}U^{(s)}_{mn}(\zeta,\bar{\zeta},\tau,\varphi) = \mathrm{e}^{2s\mathrm{i}\varphi}(1+z\bar{z})^{-s}\sqrt{\frac{C^{(s)}_m}{C^{(s)}_n}}\sum_{q=\max(0,m-n)}^{\min(m,2s-n)}\binom{n}{m-q}\binom{2s-n}{q} \\ \times(-1)^{n-m+q}\,\mathrm{e}^{\mathrm{i}n\tau}z^q\bar{z}^{n-m+q},\end{aligned} \tag{22}$$

where $C^{(s)}_n = \binom{2s}{n}^{-1}$ and $z \equiv \mathrm{e}^{\mathrm{i}\phi}\tan|\zeta|$ is now related to the stereographic projection of the two-dimensional sphere $\mathbb{S}^2 = \{\vec{v} \in \mathbb{R}^3 : \vec{v}^2 = v_0^2 + v_1^2 + v_2^2 = 1\}$ onto the complex plane, given by

$$\vec{v} = (\cos(2|\zeta|), \sin(2|\zeta|)\cos\phi, \sin(2|\zeta|)\sin\phi).$$

The finite action of $U(2)$ on annihilation (resp. creation) operators of zero modes is now

$$a_0 \to a'_0 = \mathrm{e}^{2s\mathrm{i}\varphi}(1+z\bar{z})^{-s}\sum_{n=0}^{2s}(-1)^n\binom{2s}{n}^{1/2}\mathrm{e}^{\mathrm{i}n\tau}\bar{z}^n a_n, \tag{23}$$

which leads to the polarized vacuum $|\theta'\rangle$ according to formula (14). As for the hyperbolic case, we can associate a thermal bath with the state $|\theta'\rangle$. The difference now is that the factor $|z|^2$ is unbounded from above, i.e. $|z|^2 < \infty$, which means that we have to introduce a non-zero *chemical potential* μ, such that $|z|^2 = \mathrm{e}^{(\mu-\hbar\omega)/k_B T}$, or/and allow for *negative temperatures*.

The expected value of the total energy $\hat{E}$ in the polarized vacuum $|\theta'\rangle$ proves to be

$$\begin{aligned}\langle\theta'|\hat{E}|\theta'\rangle &= |\theta|^2(1+|z|^2)^{-2s}\sum_{n=0}^{2s}\hbar\omega n\binom{2s}{n}|z|^{2n} \\ &= 2s|\theta|^2\hbar\omega\frac{|z|^2}{1+|z|^2} = 2s|\theta|^2\frac{\hbar\omega}{1+\mathrm{e}^{(\hbar\omega-\mu)/k_BT}},\end{aligned} \tag{24}$$

which is proportional to the *mean energy per mode* of the FD statistics.

4. Flat geometry and the high frequency limit

Like in the previous two cases, one can compute the finite action of the Newton–Hooke group G_0 on annihilation (resp. creation) operators of zero modes:

$$a_0 \to a_0' = \mathrm{e}^{2s\mathrm{i}\varphi}\,\mathrm{e}^{-2s|z|^2/2}\sum_{n=0}^{\infty}(-1)^n\mathrm{e}^{\mathrm{i}\tau}\sqrt{\frac{(2s)^n}{n!}}\bar{z}^n a_n, \tag{25}$$

where $z \equiv \zeta$ is unbounded, $|z| < \infty$, like for the spherical geometry. The expected value of the total energy $\hat{E}$ in the excited vacuum $|\theta'\rangle$ is now

$$\begin{aligned}\langle\theta'|\hat{E}|\theta'\rangle &= |\theta|^2\,\mathrm{e}^{-2s|z|^2}\sum_{n=0}^{\infty}\hbar\omega n\frac{(2s)^n}{n!}|z|^{2n} \\ &= 2s|\theta|^2\hbar\omega|z|^2 = 2s|\theta|^2\hbar\omega\,\mathrm{e}^{(\mu-\hbar\omega)/k_BT},\end{aligned} \tag{26}$$

which is the $\hbar\omega \gg k_BT$ limit of the BE and FD statistics. Note that, unlike for the BE and FD statistics, here we could 'reabsorb' the chemical potential μ into the vacuum expectation value of the total number of particles $|\theta|^2$ by setting $|\theta|^2 = \mathrm{e}^{-\mu/k_BT}$. Therefore, the introduction of μ in our scheme is only indispensable for the FD statistics.

5. Comments and outlook

We have separated standard ('bright') energy $\hat{H}$ from vacuum ('dark') energy $\hat{\Xi}$ in our model. However, we could combine both contributions to define a total Hamiltonian,

$$\hat{H}_{\mathrm{tot}} = \hat{H} + \frac{\kappa}{2}\hat{\Xi}, \tag{27}$$

such that the map $\hat{H} \mapsto \hat{H}_{\mathrm{tot}}$ renders the original commutation relations (1) into $[\hat{A}_+, \hat{A}_-] = 2\kappa\hat{H}$, for $\kappa = \pm 1$, and $[\hat{A}_+, \hat{A}_-] = -\hat{\Xi}$, for $\kappa = 0$. Hence, the vacuum energy is now given by

$$\langle\theta|\hat{H}_{\mathrm{tot}}|\theta\rangle = \frac{\kappa}{2}\langle\theta|\hat{\Xi}|\theta\rangle = \kappa s|\theta|^2. \tag{28}$$

Although we are dealing with a simplified (toy) model, we feel tempted to link this vacuum energy to a cosmological constant, as is done in modern cosmology. From this point of view, hyperbolic spatial geometries ($\kappa = -1$), like the anti-de Sitter spacetime, have positive pressure, which causes the expansion of empty space to slow down. In contrast, for spherical spatial geometries ($\kappa = 1$), like the de Sitter space, the expansion of empty space will tend to speed up. For flat space ($\kappa = 0$) we have zero cosmological constant. Note that we can make the parameter $|\theta|^2$ as small as we like, thus eluding huge vacuum energies for either spherical or hyperbolic geometries. Therefore, our derivation of the cosmological constant from the vacuum energy in quantum field theory is free from the traditional drawbacks of 'fine-tuning'. Moreover, the three spatial geometries, $\kappa = \pm 1, 0$, considered in this letter have different qualitative behaviour under vacuum radiation at low frequencies, something that could be used in addition to experimentally discern between these topologies.

We have presented here a quite simplified model, but we think that it grasps the essentials of more involved instances. A proper discussion of all these vacuum phenomena inside the entire conformal group $SO(4, 2)$ will be developed elsewhere.

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